Primary Mathematics Challenge – February 2022

Answers and Notes

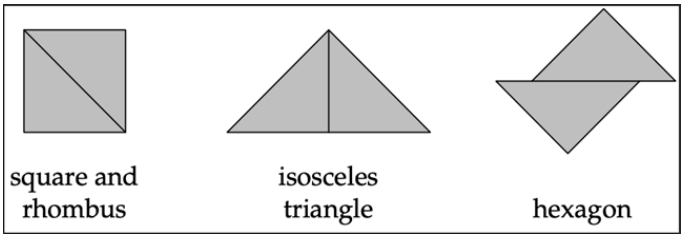
These notes provide a brief look at how the problems can be solved.

There are sometimes many ways of approaching problems, and not all can be given here.

Suggestions for further work based on some of these problems are also provided.

P1 **D** (10 minutes ÷ 2 minutes 30 seconds = 4) P2 **C** (€5.20 + €7.10 = €12.30)

1	E	48	$6 \times 8 = 48$ sprouts are needed.
2	D	9	There are three sizes of each shirt. So the striped, spotty and plain shirts each need three on display. This is a total of nine.
3	B	94 cm	The mean is $(32 + 100 + 150) \div 3 = 94$.
4	A	57	The speed is 171 million miles ÷ 3 hours = 57 million miles per hour.
5	E	30 cm	The width of the photograph is increased from 10 cm to 20 cm. This is a multiplying factor of 2. So the enlarged height of 15 cm will be $15 \times 2 = 30$ cm.
6	C	3	The largest one-digit multiple of 3 is 9. The smallest two-digit multiple of 3 is 12. The difference between 9 and 12 is 3.
7	A	A	Each girl is bowled 20 balls. Roya hits four out of five cricket balls so she hits 16 balls. Khalida hits three out of four balls, so she hits 15 balls. The true statement is that Roya hits one more than Khalida.
8	D	$\frac{1}{8}$	The big triangle would have 16 little triangles if only little triangles were used. So the fraction shaded is $\frac{2}{16} = \frac{1}{8}$.
9	A	$\frac{1}{8}$	Three-quarters of the cake is to be divided into six equal slices. This is $\frac{3}{4} \div 6 = \frac{3}{24} = \frac{1}{8}$.
10	D	equilateral triangle	The diagrams below show how one can form a square, a rhombus, an isosceles triangle and a hexagon from congruent right-angled triangles:



Is a rhombus a square? Yes, a rhombus is both square and rhombic in shape. A female teacher can be a woman, a teacher and also a mathematician!

11	B	0.75	The weekend lost 250 ml. In the five weekdays $5 \times 100 = 500$ ml were lost. This gives a total of 750 ml = 0.75 l.
12	C	24	I am looking for when my mother's age is twice that of mine. As time passes and adding years on, our ages become:

My age	14	16	18	20	22	24
Mother's age	38	40	42	44	46	48

So I am now 24 years old.

13	D	80 cm^2	The smaller square touches the sides of the large square leaving the triangles with base 8 cm and height 4 cm. The area of one triangle is $\frac{1}{2} \times 4 = 16$ cm^2. The area of the larger square is $12 \times 12 = 144$ cm^2. So the area of the smaller square is $144 - 4 \times 16 = 80$ cm^2
14	E	$\frac{44+44}{44}$	A gives $2 + 2 = 4$. B gives $64 \div 16 = 4$. C gives $16 \div 4 = 4$. D cancels down to 4. E gives $88 \div 44 = 2$.

15	C	12	If Gregory has n onions, he has $2n$ carrots and $4n$ potatoes. So $7n = 84$. So the number of onions is 12.
16	D	64%	After one translation there will be 80% meaning left. After the next year there will only be 80% of 80% = 64%.
17	D	$19\,m^2$	The total area of the front and back faces is $2 \times 3 \times 2 = 12\ m^2$. The total area of the two ends is $2 \times 2 \times 1 = 4\,m^2$. The area of the base is $3 \times 1 = 3\,m^2$. Adding, the total area of metal used is $12 + 4 + 3 = 19\,m^2$.
18	C	$2n - 1$	Number of days n 1 2 3 4 5 Number of press-ups: 1 3 5 7 9 The pattern is to take twice the numbers of days minus one to get the number of press-ups; i.e. $2n - 1$.
19	D	2 : 3	If the mid-points of the three sides of triangle ABC are all joined by straight lines, four small triangles are created. There will be six of these small triangles in the hexagon. So the ratio is 4 : 6 which is 2 : 3.
20	E	240 km	The length of the minute hand is not important here. The tip traces out 27 m each minute. That is $27 \times 24 \times 365$ m each year. Approximating, levelling some numbers up and others down, an estimate for the distance travelled each year is: $30 \times 20 \times 350 = 600 \times 350 = 210$ km. Looking at the responses, the nearest is 240 km every year. More accurately, it travels $(27 \times 24 \times 365) \div 1000 = 236.52$ km.
21	B	2	Jenny has 20 friends, but 18 of them work in a charity shop. Some of these can be helping children to read and helping with animals. So the largest number not doing any of these is 2.
22	D	1 hours 45 mins	Using the number 144 gives difficult fractions and is best avoided if possible. However, it takes Joe 140 minutes to complete the task, therefore he does $\frac{1}{140}$ of the task per minute. Together they take 60 minutes, so Joe and Josie take $\frac{1}{60}$ of the task per minute. This means that Josie must complete $\frac{1}{60} - \frac{1}{140} = \frac{1}{105}$ of the task per minute, so she takes 105 minutes to complete the task. Alternatively, the lowest common multiple of 60 and 140 is 420. In 420 minutes Joe sharpens 3 boxes, and together they would sharpen 7 boxes. On her own, therefore, Josie would sharpen 4 boxes. So she takes 420 minutes to sharpen 4 boxes and therefore 105 minutes to sharpen one box.
23	D	7.12 a.m.	The French had 10 hours for our 24 hours. So a French hour has the same length of time as $24 \div 10 = 2.4$ hours. Three French hours therefore equals $3 \times 2.4 = 7.2$ hours. Now 0.2 of an hour is $0.2 \times 60 = 12$ minutes. So the time by our clocks would be 7.12 a.m.
24	A	0	The numbers are: 789, 987, 456, 654, 123, 321, 741, 147, 852, 258, 963, 369, 753, 357, 159 and 951. They all have a digit sum which is a multiple of 3. So they are all multiples of 3 and therefore there are no prime numbers.
25	E	8	For this number, $3 \times (7 + 0 + 0 + 5 + 8 + n) + (9 + 8 + 9 + 6 + 8 + 9 + 7) = 60 + 3 \times n + 56 = 116 + 3 \times n$ which must come to a multiple of 10. For n to be a whole number, we have to add on 24 to get 140. So $n = 8$.

Some notes and possibilities for further problems

P2 Pupils can work out money problems in most currencies in the world. Here are three:

a) \$4.60 + \$3.20 b) Kr. 2.78−Kr 1.18 c) ¥3.23+¥5.78.

There are two currencies in the world whose division units are not based on a power of ten: they are the Mauritanian *ouguiya* and the Malagasy *ariary*.

2 Suppose there were four designs of shirt, each made in three sizes. How many hangers would be needed now?

4 Sometimes the planet Pluto is about 4.67 billion miles from Earth. How long will signals take to get from Pluto to Earth? (4.67 billion miles is 4670 million miles.)

5 An enlargement of 2 is easy. Here is a slightly harder problem: a photo measuring 10 cm×15 cm is enlarged so that the image similar but enlarged. The width of the larger photo is 25 cm, what is the height?

A 30 cm B 32 cm C 37.5 cm D 40 cm E 45 cm

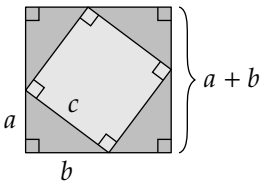
10 How many shapes can be made with:

a) two identical equilateral triangles?
b) two identical right-angled triangles (not isosceles)?
c) two identical squares?

12 Algebra can be used. If n is the number of years between then and now, my age is $14 + n$, and my mother's age is $38 + n$. Then $2(14 + n) = 38 + n$, and therefore $28 + 2n = 38 + n$: so $n = 10$. So I am now $14 + n = 24$ years old.

13 If 3 cm and 4 cm are the lengths of two sides of the small triangles (not the hypotenuse) then we can calculate the length of the hypotenuse as 5 cm, by using Pythagoras' Theorem.

We can then generalize the calculation to prove Pythagoras' Theorem as follows: the area of the large square is $(a + b)^2$. The area of the small square is c^2. The area of one small triangle is $\frac{1}{2}(a \times b)$. So we can calculate the area of the small square as: $(a + b)^2 - 4 \times \frac{1}{2}(a \times b) = a^2 + 2ab + b^2 - 2ab = a^2 + b^2$. In the small square therefore, $c^2 = a^2 + b^2$ so $c = \sqrt{a^2 + b^2}$, which demonstrates Pythagoras' Theorem.



14 A famous problem is the *Four Fours Puzzle*: use exactly four 4s to form every integer from 0 to 50, using only the operators $+$, $-$, $\times$, $\div$, () (brackets), . (decimal point), $\sqrt{}$ (square root) and ! (factorial). For example. $44 - 44 = 0$ or $4! + \frac{4}{\sqrt{4}} = 26$. How many other numbers can your pupils make?

18 In this problem, the number of press-ups increases by 2 each day, so there is likely to be a 2 in the formula. Try these:

a) Frank says he can cycle even further every day. One day he cycles 6 miles.
Every day after this he increases his distance by 4 miles.
What is the formula for how far he cycles on day n?

b) The water in a tank is leaking. The first day there are 100 litres of water in the tank.
On the second day, 1 litre leaks out, on the third day 2 litres leak out, and so on.
How much water is in the tank after n days?

22 This problem is interesting in that there are several ways to think about it. If the number of pencils had been 140, then Joe would sharpening at a rate of one per minute. This gives an easier way to solve the problem than with 144 pencils. But then we didn't want this problem to be easy!

23 What time would it be in the UK if the French clock showed: a) 5 hr? b) 9 hr?
If the UK time was 3 pm, what time would it be on a French clock?
French Revolutionary Time officially began on 24 November 1793. The French manufactured clocks and watches showing both decimal time and standard time on their faces (allowing for both conversion and confusion). These clock faces were spectacularly weird. The system proved unpopular. The French officially stopped using decimal time after just 17 months.

The first watch shown here has both 10-hour and 12-hour markings.
The second clock face has both 10-hour and 24-hour markings.

25 Pupils could choose their favourite book and do the check on the ISBN number.
Which book has this ISBN number: 978-0747532699 ?